A Common Basis for the Correlation of Forced and Natural Convection to Horizontal Cylinders

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Natural and forced convection have always been correlated on different bases although the two phenomena have much in common. Accordingly, a method for correlating both types of convection on the same basis, at least for heat transfer outside horizontal cylinders, would be of interest.

A tentative method of accomplishing this for transverse convection is presented, involving certain simplifying assumptions relating to drag and buoyancy, from which an "effective velocity" for natural convection is calculated (by means of the well-known drag correlation) and incorporated in a Reynolds number. The Nusselt number for natural convection is then correlated in terms of the Prandtl number and this Reynolds number in much the same manner as that for McAdams's well-known correlation for forced convection. For 150 different combinations of independent variables covering seven different fluids and wide ranges of diameter, surface temperature, and bulk fluid temperature, the transformed natural-convection data agree with Douglas and Churchill's recent refinement of McAdams's relationship for forced convection with an average deviation of about ±10%.

The methods for correlating forced and natural convective heat transfer differ. For forced convection the Nusselt number Nu is generally presented as a function of the Prandtl number Pr and the Reynolds number Re, and for natural convection Nu is generally correlated as a function of Pr and the Grashof number Gr. Qualitatively speaking, however, the two types of convective heat transfer have much in common. It is therefore of interest to investigate the possibility of devising a method whereby both forced and natural convection can be correlated on the same basis.

One possible scheme is to apply the method of correlating forced convection, namely expressing Nu as a function of Pr and Re, to natural convection as well. This can be accomplished if a suitable effective velocity for natural convection can be obtained for substitution in the Reynolds number. For transverse convection outside horizontal cylinders Brown and Marco (*1*) assumed that the moving fluid film in natural convection uniformly accelerates over a distance of one cylinder diameter D. This assumption, of course, completely neglects the drag of the moving fluid against the stationary fluid. No experimental check is presented although Jakob (*4*) indicates that the analogous assumption for vertical surfaces yields results that agree in their order of magnitude with experimental data.

THEORY

Accordingly, in an effort to include the effect of drag, the following set of simplifying assumptions is tentatively proposed for horizontal cylinders.

1. The boundary layer surrounding the body is the fictive film of Langmuir (*5*), defined as a uniform layer of fluid with a thickness B sufficient to account by conduction alone for all the nonradiative heat transferred. Thus the fictive film has an outside diameter of $D + 2B$, as shown in Figure 1.
2. This hot fictive fluid cylinder rises under a buoyant force equal to that exerted on a cylinder of diameter $D + 2B$ filled with fluid which is at the arithmetic mean film temperature t_f, defined as half the sum of the bulk fluid temperature t_0 and the temperature at the surface of the solid t_s.
3. The fluid film rises at a constant velocity u.
4. The drag resistance is that for a cylinder of diameter $D + 2B$ moving at this velocity through the bulk fluid.

These assumptions are admittedly only simplifying approximations to the real situation. Nevertheless, by means of them, a wide range of natural-convection data for horizontal cylinders was converted (*3*) into the same form as forced-convection data and correlated in the same way. The source of the natural-convection data was McAdams's generalized dimensionless correlation (*7*), which represents the smoothed experimental results of many investigations covering a wide variety of conditions. Various combinations of fluid, D, t_0, and t_s, were hypothesized, and Nu was computed for each combination using this generalized correlation for natural convection. The fluids considered included the gases air, carbon dioxide, and ethane and the liquids water, aniline, *n*-octane, and *m*-xylene. D ranged from 0.01 to 10 in,.

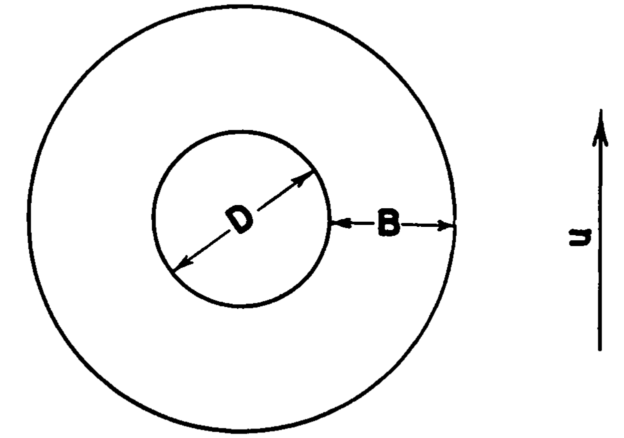

Fig. 1. Cylinder surrounded by fictive film.

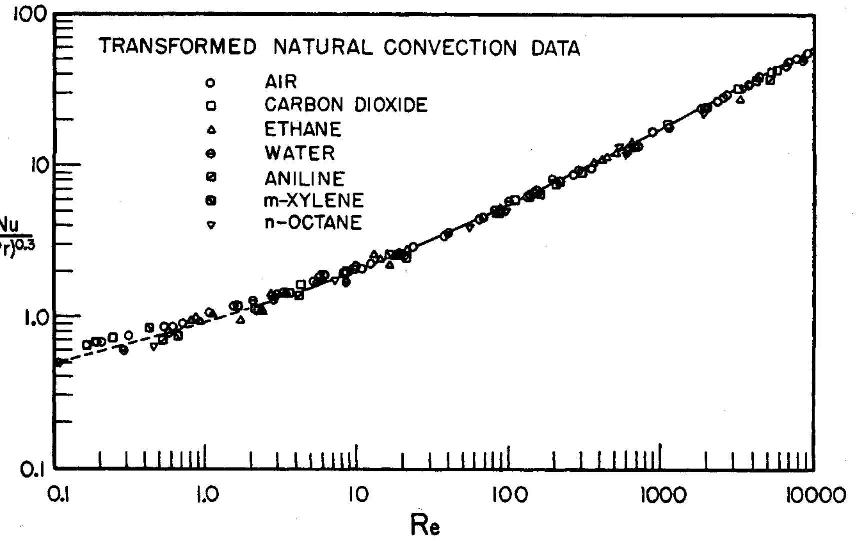

Fig. 2. Comparison of transformed natural-convection data with a generalization (to include *Pr*) of the forced-convection correlation of Douglas and Churchill.

t_0 ranged from 0° to 1,000°F., and $\Delta t = t_s - t_0$ ranged from 1° to 1,000°F. *Pr* ranged from 0.71 to 0.78 for the gases, and from 1.9 to 11.4 for the liquids. The fictive film thickness was then computed from the expression

$$B = \frac{D}{2}\left(e^{2/Nu} - 1\right) \qquad (1)$$

which follows (*6*) from the definition of *B*.

The forces acting on a body moving at constant velocity must balance; in other words, the net upward buoyant force F_b acting on the rising film must balance the downward drag force F_d. Per unit length of cylinder, this upward force is given by

$$F_b = \frac{\pi}{4}(D + 2B)^2(\rho_0 - \rho_f)g \qquad (2)$$

where g is the acceleration due to gravity and ρ_0 and ρ_f are the densities of the fluid evaluated at t_0 and t_f respectively. The downward force per unit length can be expressed in terms of the drag coefficient by

$$F_d = \tfrac{1}{2}C\rho_0 u^2(D + 2B) \qquad (3)$$

Perry (*10*) gives C as a function of the Reynolds number $Re' = (D + 2B)u\rho_0/\mu_0$, where μ_0 is the viscosity at t_0. Combining Equations (2) and (3) and incorporating Re' into the result yields

$$C(Re')^2 = \frac{\pi(D + 2B)^3(\rho_0 - \rho_f)g\rho_0}{2\mu_0^2} \qquad (4)$$

Accordingly, by substitution of *D*, *B*, and the appropriate fluid properties into Equation (4) the quantity $C\,(Re')^2$ was calculated. From this, Re' itself was determined from the aforementioned drag correlation replotted in the form $C\,(Re')^2$ vs. Re'. From Re' the effective velocity u was computed. The procedure of using a replot was applied merely to eliminate trial-and-error solutions for u. Next the Reynolds number for heat transfer $Re = Du\rho_f/\mu_f$, where μ_f is the viscosity at t_f, was computed. Finally *Nu* was plotted as a function of *Pr* and *Re* in accordance with a generalization (*9*) (merely to include *Pr*) of Douglas and Churchill's recent refinement (*2*) of McAdams's generalized correlation for forced convection (*8*) and compared with the correlation curve itself.

DISCUSSION

Figure 2 shows this comparison. The generalization of Douglas and Churchill's curve for forced convection fits the transformed natural-convection data with an average deviation of about ±10%. This is almost as good as the fit between the curve itself and the experimental forced-convection data (not shown here) on which it is based. In view of the very wide range of variables involved, this is considered very satisfactory agreement. Thus these simplifying assumptions make possible the correlation of natural convection on the same basis as forced convection, at least for horizontal cylinders.

If application of Figure 2 to an actual calculation in natural convection is desired, say to find *Nu*, a simple trial procedure is employed, as follows. *Nu* is assumed, *B* calculated from Equation (1), $C\,(Re')^2$ computed from Equation (4), Re' determined from the replot of the drag correlation mentioned above, *Re* itself calculated, and *Nu* obtained from Figure 2, the entire procedure being repeated as necessary.

By incorporating the appropriate modifications in the derivation to account for the difference in geometry, the authors have attempted to extend the principle to spheres but so far with less success, perhaps owing to greater deformation. A modification in assumption 2, namely consideration of the fluid cylinder as being hollow with an internal diameter of *D*, has been studied as well. For cylinders this has proved somewhat less satisfactory than the original assumption although so far it appears fairly satisfactory for spheres. Further work on the entire subject is planned.

CONCLUSION

By means of suitable assumptions involving buoyancy and drag, data for natural convection can be transformed and correlated on the same basis as forced convection, at least for heat transfer outside horizontal cylinders.

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NOTATION

- B = thickness of fictive film, ft.
- C = drag coefficient, dimensionless
- D = diameter of solid cylinder, ft.
- e = base of natural logarithms, dimensionless
- F = force per unit length, poundals/ft.
- Gr = Grashof number, dimensionless
- g = gravitational acceleration, ft./sec.^{2}
- Nu = Nusselt number, dimensionless
- Pr = Prandtl number, dimensionless
- Re, Re' = Reynolds number for the solid cylinder and the fluid cylinder respectively, dimensionless
- t = temperature, °F.
- Δt = difference in temperature between the surface of the solid and the bulk fluid, °F.
- u = effective velocity, ft./sec.
- μ = viscosity, lb./(ft.)(sec.)
- ρ = density, lb./cu. ft.

Subscripts

- b = buoyant
- d = drag
- f = average for film
- 0 = bulk
- s = solid surface

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